Experiment Record: Conceptualization of Random Error and Propagation of Error

Experiment Title

Conceptualization of Random Error and Propagation of Error by Measuring the Dimensions of a Thin Metallic ball Using Screw Gauge and Vernier Caliper

Objective

1. To measure the dimensions (diameter and length) of a thin metallic ball using a screw gauge and Vernier caliper.
2. To calculate the volume and surface area of the metallic ball.
3. To understand random errors and analyze the propagation of errors in measurements.

Apparatus Required

- Screw gauge
- Vernier caliper
- Thin metallic ball
- Calculator

Theory

Measurement Tools

- **Screw Gauge:** Measures diameter with high precision.

$$L.C. = \frac{\text{Pitch of screw}}{\text{Number of divisions on circular scale}}$$

- **Vernier Caliper:** Measures length.

  $L.C.$ = Value of 1 Main Scale Division (MSD)−Value of 1 Vernier Scale Division (VSD)

  Example: If 10 Vernier scale divisions = 9 main scale divisions, then

$$L.C. = 1\,\text{mm} - 0.9\,\text{mm} = 0.1\,\text{mm}.$$

Formulas

- **Volume of ball:**

$$V = 4\pi r^3/3$$

  where r = radius of ball.

- **Surface Area of ball:**

$$A = 4\pi r^2$$

Error Propagation

When we measure a set of data, We use standard deviation to estimate error.

$$\sigma_x = \sqrt{(x_i - \overline{x_i})^2}$$

- For **multiplication or division**, relative errors are added:

$$\frac{\sigma_z}{z} = \frac{\sigma_x}{x} + \frac{\sigma_y}{y}$$

- For **addition or subtraction**, absolute errors are added:

$$\sigma_z = \sigma_x + \sigma_y$$

- Errors in this experiment:
  - **Volume Error:**

$$\sigma_V = 4\,\pi\,r^2\,\sigma_r$$

    Surface Area Error:

$$\sigma_A = 8\,\pi r^2\,\sigma_r$$

Procedure

1. **Measuring Diameter:**
   - Place the metallic ball in the jaws of the screw gauge.
   - Take three readings of the diameter (d) at different points along the ball.
   - Calculate the mean diameter and derive the radius ($r = d/2$).
2. **Calculations:**
   - Compute the volume and surface area using the measured dimensions.
   - Use the formulas for error propagation to calculate the uncertainties in the volume and surface area.

Observations

zero coincidence =
zero error =
zero correction =

Screw Gauge Readings

Obs.	**MSR**	**HSR**	**Corrected HSR**	**Diameter (mm)** (MSR + Corrected HSR × LC)	**Average Diameter** ($\overline{x_i}$)	**Deviation**^{2} $(x_i - \overline{x_i})^2$	$\sigma_{\textbf{Diameter}} = \sqrt{\overline{(x_i - \overline{x_i})^2}}$
1							
2							
3							
⋮							

Mean Diameter: $\bar{d} = \ldots \pm \sigma_d$

Vernier Caliper Readings

Obs.	**MSR**	**VSR**	**Diameter (mm)** (MSR + VSR × LC)	**Average Diameter** ($\overline{x_i}$)	**Deviation**^{2} $(x_i - \overline{x_i})^2$	$\sigma_{\textbf{Diameter}} = \sqrt{\overline{(x_i - \overline{x_i})^2}}$
1						
2						
3						
⋮						

Calculation of Standard Deviation for Volume of a Sphere

Volume formula: $V = \frac{4}{3}\pi r^3$, where $r = \frac{d}{2}$

0.1 Using screw Gauge

Obs.	Diameter d (mm)	Radius $r = \frac{d}{2}$	Volume $V = \frac{4}{3}\pi r^3$ ($\mathbf{mm^3}$)	$V - \overline{V}$	$(V - \overline{V})^2$
1					
2					
3					
4					
5					
Mean Volume $\overline{V}$			$= \ldots$		
Standard Deviation $\sigma_V = \sqrt{(V - \overline{V})^2}$					$= \ldots$

0.2 Using Vernier Callipers

Obs.	Diameter d (mm)	Radius $r = \frac{d}{2}$	Volume $V = \frac{4}{3}\pi r^3$ ($\mathbf{mm^3}$)	$V - \overline{V}$	$(V - \overline{V})^2$
1					
2					
3					
4					
5					
Mean Volume $\overline{V}$			$= \ldots$		
Standard Deviation $\sigma_V = \sqrt{(V - \overline{V})^2}$					$= \ldots$

Calculations

Volume

$$V = 4\pi r^3/3$$

Absolute error in volume:

$$\sigma_V = 4\pi r^2 \sigma_r$$

Surface Area

$$A = 4\pi r^2$$

Absolute error in surface area:

$$\sigma_A = 8\pi r \sigma_r$$

Results

1. **Dimensions of the ball:**
   - Diameter (d): $\ldots \pm \ldots$ mm
   - Length (h): $\ldots \pm \ldots$ mm
2. **Calculated Quantities:**
   - Volume (V): $\ldots \pm \ldots$ mm^3
   - Surface Area (A): $\ldots \pm \ldots$ mm^2
3. **Error Analysis:**
   - Propagated error in volume: ...
   - Propagated error in surface area: ...

Conclusion

The dimensions of the metallic ball were successfully measured using a screw gauge and Vernier caliper. The calculated volume and surface area were found with propagated uncertainties, providing insight into the impact of random measurement errors.

1 Not required for record or exam

Explanation of Least Count (L.C.) of Vernier Caliper

The **Least Count (L.C.)** of a Vernier caliper is the smallest value that can be measured accurately using the instrument. It is derived based on the difference between the values of one main scale division (MSD) and one Vernier scale division (VSD).

Formula for Least Count

$L.C. = \text{Value of 1 Main Scale Division (MSD)} - \text{Value of 1 Vernier Scale Division (VSD)}$

Understanding the Vernier Scale Division (VSD)

The Vernier scale has a specific number of divisions that correspond to a fixed length on the main scale. For example:

- If 10 Vernier scale divisions = 9 main scale divisions,
- Then, the value of 1 Vernier scale division is:

$$\text{VSD} = \frac{\text{Value of 1 Main Scale Division (MSD)}}{\text{Number of Vernier Scale Divisions}}$$

  Substituting the values:

$$\text{VSD} = \frac{1\,\text{mm}}{10} = 0.1\,\text{mm}$$

Calculating the Least Count

The least count is then calculated as:

$$L.C. = \text{MSD} - \text{VSD}$$

If:

- MSD = 1 mm,
- VSD = 0.9 mm (since 10 Vernier divisions = 9 main scale divisions),

Then:

$$L.C. = 1\,\text{mm} - 0.9\,\text{mm} = 0.1\,\text{mm}$$

This means the Vernier caliper can measure with a precision of 0.1 mm.

Conclusion

The least count of a Vernier caliper is the smallest measurable value and is derived using:

$L.C.$ = Value of 1 Main Scale Division (MSD)−Value of 1 Vernier Scale Division (VSD).

This method ensures accurate understanding of how the main scale and Vernier scale interplay to achieve fine measurements.